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Parametric analysis of direct contact evaporation process in a bubble column

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Abstract—This paper presents a detailed analysis of the influence of several important parameters, such as the breakage of droplets, the driving temperature difference, the initial mean diameter of drops from the bottom distributor, and the maximum diameter of stable drobbles on the direct-contact evaporation process in a bubble column. Some simulation results agree with experimental data reasonably well. Numerical calculations show that there is an approximately proportional relationship between the evaporation height and the reciprocal of the driving temperature difference. © 1998 Elsevier Science Ltd. All rights reserved.

INTRODUCTION

Direct-contact heat transfer with a dispersed population of evaporating droplets in another immiscible liquid has a special advantage if an intensive cooling of a medium is required when there is only a small driving temperature difference between the continuous and the dispersed phase. It is widely used in many industrial processes, such as water desalination and purification [1], spray cooling [2], spray combustion [3], crystallization [4], solar, geothermal and ocean-thermal power [5, 6], and thermal energy storage systems [7]. Because a great mass of heat can be exchanged, this form of heat transfer is an important way to prevent thermal explosions (reaction runaway) in chemical reactors where strong exothermic chemical reactions exist [8].

The heat transfer efficiency depends strongly on the interfacial surface between the continuous and the dispersed phase. In spite of the same hold-up the total interfacial surface available for heat exchange can be very different for different size distributions of the dispersed phase. In order to estimate the characteristics and properties of the dispersed phase, and even the overall performance of a dispersion system, we are required to have a good knowledge of not only the hold-up but also the size distribution of the dispersed droplets. In the literature the researchers in this area have paid much more attention to the former than to the latter. Core and Mulligan [9] presented a population balance model to discuss the heat transfer characteristics of direct contact evaporation in batch reactors. Steiner [10] studied a similar problem in bubble columns. However, the breakage and the coalescence of dispersed droplets were not included in their models.

The authors proposed a mathematical–physical model to predict the statistical evaporation height of a droplet swarm in bubble columns [11]. In this model the breakage and the coalescence of dispersed droplets could be included. This paper presents a detailed analysis of the influence of several important parameters, such as the breakage of droplets, the temperature difference between the continuous and the dispersed phase, the initial mean diameter of drops from the bottom distributor, and the maximum diameter of stable drobbles on the statistical evaporation height and the density distribution of dispersed droplets in a bubble column. Due to the low number density of particles in the present problem, coalescence has little influence on the evaporation process and is hence assumed negligible.

DESCRIPTION OF THE PROBLEM

Consider a bubble column up which flows a liquid, called the continuous phase. A population of drops, called the dispersed phase, with a certain size distribution enter this column through a distributor at the bottom, and begin evaporating and thus withdrawing heat from the continuous phase. The latter will be cooled. In general, the continuous phase is assumed to be uniform and immiscible with the vapour and unevaporated liquid of the dispersed phase. As the dispersed droplets complete their evaporation, the vapour exits from the free surface of the continuous liquid and is subsequently withdrawn from the head part of this column.

Studies on the evaporation of a single droplet were carried out by various investigators, considering different aspects of the problem [12–17]. The evap-

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NOMENCLATURE

$b(x)$	breakage probability function [s^{-1}]
D	drobble diameter [m]
D_{max}	maximum diameter of stable drobbles [m]
g	acceleration of gravity [m s^{-2}]
h	column height [m]
h_f	heat transfer coefficient [W m^{-2} K^{-1}]
k	constant in Mersmann's formula
k_v	volumetric heat transfer coefficient [W m^{-3} K^{-1}]
n	number density of drobbles [m^{-4}]
p	pressure [Pa]
$p(y, x)$	distribution function of daughter drobbles [m^{-3}]
q	growth rate of drobbles with column height
$\dot{Q}$	heat transferred per unit time [W]
t	time [s]
T	temperature [K]
v	rising velocity of drobbles [m s^{-1}]
V	volume [m^3]
w	growth rate of drobbles with time [m s^{-1}]
x	volume of a mother drobble [m^3]
y	volume of a daughter drobble [m^3]
$\bar{y}$	average volume of daughter drobbles [m^3].

Greek symbols

Δh_v	evaporation enthalpy of the dispersed phase [J kg^{-1}]
ε	vapour hold-up
ν	number of daughter drobbles
ρ	density [kg m^{-3}]
σ	surface tension [N m^{-1}]
σ_y	standard deviation of the distribution $p(y, x)$.

Subscripts

0	initial value
C	continuous phase
D	dispersed phase
g	gas
l	liquid
i	inlet
o	outlet.

oration of a droplet is possible provided that the temperature of the continuous phase is higher than the boiling point of the dispersed phase. In the course of evaporation the droplet forms a gas–liquid two-phase particle. This particle has been named "Drobble" (Drop + Bubble) by Sudhoff et al. [18]. Mori [19] presented an excellent discussion on the possible configurations of a drobble. The most probable configuration is shown in Fig. 1 where the surface tension of the dispersed droplets is greater than that of the continuous liquid. In this figure, 2β is called the evaporation open angle. In general, a drobble is characterized by the evaporation degree, defined as the mass ratio of the vapour portion of a particle to the total particle. It is a monotonically increasing function of the open angle.

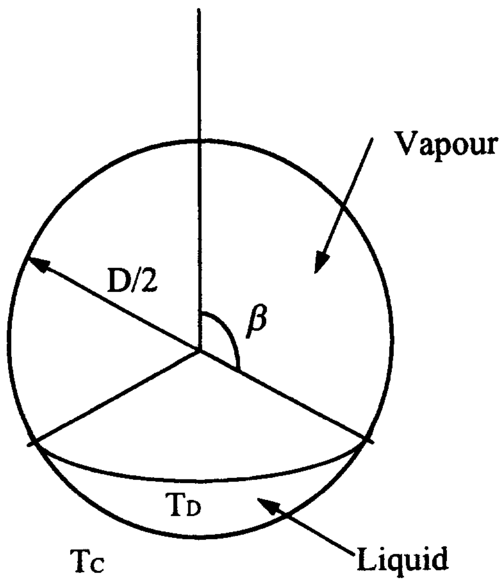

Fig. 1. Configuration of a drobble.

The evaporation process of a droplet is very complicated. According to experiments [20] it can be divided into approximately five stages based on the evaporation degree:

1. evaporation degree = 0 (initial state): spherical drop;
2. evaporation degree > about 0.1%: spherical drobble;
3. evaporation degree > about 1%: ellipsoidal drobble, in which the unevaporated liquid takes the form of film in the lower part of this particle, shown in Fig. 1;
4. evaporation degree > about 10%: umbrella-shaped drobble, the shape of which cannot be described by a simple mathematical formula; and
5. evaporation degree = 100% (end state): oblate bubble.

There are fewer theoretical studies on the evaporation

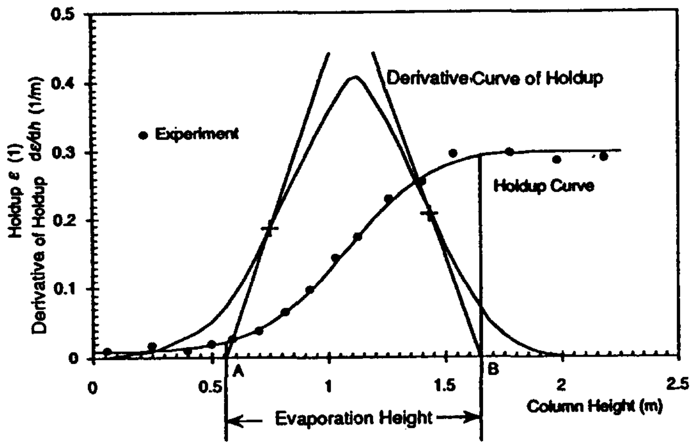

Fig. 2. Experimental determination of the evaporation height.

of a drop swarm because interactions such as the coalescence and the breakage of droplets are difficult to describe in both mathematics and physics. It is impossible to experimentally determine the area of interfacial surfaces available for heat transfer between the two phases. The volumetric heat transfer coefficient, defined as

$$k_v = \frac{\dot{Q}}{V \cdot \Delta T} \tag{1}$$

is usually used to describe the characteristics of heat exchange in direct-contact evaporation systems by researchers [9, 10, 21–23]. In equation (1) $\dot{Q}$ is the heat removed from the continuous phase per unit time due to evaporation, V is the total volume of both phases, and ΔT is the driving temperature difference between the two phases. The logarithmic mean temperature difference is taken for this definition, that is

$$\Delta T = \frac{(T_{Co} - T_{Do}) - (T_{Ci} - T_{Di})}{\ln \dfrac{T_{Co} - T_{Do}}{T_{Ci} - T_{Di}}}. \tag{2}$$

It is understood that the smaller droplets complete their evaporation earlier than the bigger ones. It is necessary to define a quantity to represent this characteristic of evaporation from the statistical aspect. Hence the evaporation height is introduced, theoretically defined as the column height over the bottom distributor at which a certain mass fraction (near 1.0) of all droplets has been evaporated. This quantity is one of the most important design parameters in engineering.

The evaporation height cannot be determined directly from experiments. An effective indirect method was designed by Nitsche [22]. Neglecting the friction between the medium and the wall, a relationship between the vapour hold-up and the pressure gradient is readily derived by considering the momentum balance in a differential volume element

$$\varepsilon(h) = \frac{1 + \dfrac{1}{\rho_l g} \cdot \dfrac{dp}{dh}}{1 - \dfrac{\rho_g}{\rho_l}}. \tag{3}$$

The curves of ε vs h and $d\varepsilon/dh$ vs h can be drawn from the measured pressure profile along the column height. A typical example is shown in Fig. 2. From the curve of $d\varepsilon/dh$ vs h we can find two inflexion points, marked "+" in Fig. 2. If we make tangents to the curve at the two marked points and mark the two intersection points of these two tangents with the h abscissa as A and B, the length between A and B is considered the evaporation height.

With this method Nitsche [22] and Steiner [10] performed experiments on different columns and obtained a lot of reliable experimental data.

MODEL

A particulate breakage system can be described by the population balance equation (PBE) in mathematics. For the present problem, the PBE is written as

$$\frac{\partial(vn)}{\partial h} + \frac{\partial(wn)}{\partial D} = \int_{\frac{\pi}{6}D^3}^{\infty} v(x') b(x') p\left(\frac{\pi}{6}D^3, x'\right) n(x', t)\, dx' - b\left(\frac{\pi}{6}D^3\right) n\left(\frac{\pi}{6}D^3, t\right) \tag{4}$$

where n is the number of drobbles per unit drobble diameter per unit volume, $p(x, x')$ is the breakage function of a drobble with volume x', $b(x)$ is the breakage frequency of a drobble, $v(x)$ is the number of daughter drobbles per breakage, and $v = (dh/dt)$ and $w = (dD/dt)$ are the rising velocity and the growth rate of drobbles, respectively. From the energy conservation we can derive the growth rate of a drobble:

$$w = \frac{2h_f(T_C - T_D)(\rho_l - \rho_g)}{\rho_l \rho_g \Delta h_v} \tag{5}$$

where h_f is the heat transfer coefficient and can be determined from the Nusselt number. A correction formula for the Nusselt number by the authors [24] is used in modelling.

The rising velocity of drobbles by Raina *et al.* [25] with the correction from Gal-Or and Walatka [26] gives

$$v = 1.91\left\{\left[1 - \frac{\rho_D}{\rho_C}\left(\frac{D_0}{D}\right)^3\right]\frac{\sigma}{\rho_C D}\right\}^{1/2}\frac{(1-\varepsilon)^2}{1-\varepsilon^{5/3}}. \tag{6}$$

In our simulation we model two questions about the breakage of drobbles: when does a drobble break up and how does this drobble break up. Mersmann's formula for the maximum diameter of stable dispersed droplets is utilized [27, 28]:

$$D_{\max} = \sqrt{\frac{k\sigma}{(\rho_C - \rho_D)g}} \tag{7}$$

where k is a constant of around 9.0. A drobble will break up once its diameter reaches this maximum value.

As analyzed by Chatzi *et al.* [29], the breakage process cannot be completely described if we do not know the size distribution of the daughter drobbles resulting from the breakage of a mother drobble, $p(y, x)$, and the number of the daughter drobbles per breakage, $v(x)$. Multiple breakages of a drobble are possible, but the breakage is most likely binary [29–31], that is,

$$v(x) = 2. \tag{8}$$

Then the average volume of the two daughter drobbles from a mother drobble of volume x is

$$\bar{y} = \frac{x}{2}. \tag{9}$$

Consequently, the size distribution of the daughter drobbles should be symmetrical about $y = \bar{y}$, because two drobbles of diameter

$$y_1 = \frac{\bar{y}}{2} + \left(y_1 - \frac{\bar{y}}{2}\right) \quad \text{and} \quad y_2 = \frac{\bar{y}}{2} - \left(y_1 - \frac{\bar{y}}{2}\right)$$

always appear simultaneously. The breakage distribution is reasonably assumed to be normal while the breakage process can be considered as the combined result of a large number of independent random events [29]

$$p(x, y) = \frac{1}{\sqrt{2\pi}\sigma_y} e^{-\frac{(y-\bar{y})^2}{2\sigma_y^2}}. \tag{10}$$

Coulaloglou and Tavlarides [30] and Chatzi *et al.* [29] chose the standard deviation σ_y in such a way that about 99.6% of the daughter drobbles lie in the interval $[0, x]$, that is,

$$\sigma_y = \frac{\bar{y}}{3} = \frac{x}{6}. \tag{11}$$

Instead, we take

$$\sigma_y = \frac{\bar{y}}{3.5} = \frac{x}{7} \tag{12}$$

so that about 99.95% of the daughter drobbles lie in that interval.

The log-normal distribution of drops from the bottom distributor is assumed as the initial condition.

SOLUTION METHOD

In numerical calculation, the population of drobbles is divided into M classes, and the height of the column is discretized into N cells.

We recently proposed an effective new method, named *splitting + analog*, to solve the PBE with particle size growth [32]. In the present problem, the equation is *split* into two substeps, that is, the growth step and the breakage step, within a discretized space step:

$$\begin{cases} \dfrac{1}{2}\dfrac{\partial(vn)}{\partial h} = -\dfrac{\partial(wn)}{\partial D} & h^n < h \leqslant h^n + \dfrac{1}{2}\Delta h \\ \dfrac{1}{2}\dfrac{\partial(vn)}{\partial h} = \displaystyle\int_{\frac{\pi}{6}D^3}^{\infty} v(x')b(x') & \\ \quad \times p\left(\dfrac{\pi}{6}D^3, x'\right)n(x', t)\,\mathrm{d}x' & \\ \quad -b\left(\dfrac{\pi}{6}D^3\right)n\left(\dfrac{\pi}{6}D^3, t\right) & h^n + \dfrac{1}{2}\Delta h < h \leqslant h^{n+1} \end{cases} . \tag{13}$$

For the first substep the discretization equations are formed through an *analog* with the one-dimensional continuity equation in fluid dynamics as

$$(vn)^n = (vn)^0 \frac{(\Delta D)^0}{(\Delta D)^n} \tag{14}$$

$$(\Delta D)^n = (\Delta D)^0 = \int_0^{h^n} \Delta q \, \mathrm{d}h \tag{15}$$

where

$$q = \frac{w}{v} \tag{16}$$

called the growth rate of drobbles with the column height.

PARAMETRIC ANALYSIS

Breakage of drobbles

The breakage of drobbles is the most important characteristic of the present dispersion system. Core and Mulligan [9] and Steiner [10] studied heat transfer and population characteristics of dispersed evaporating droplets, but their disregarding of the breakage of droplets limits their models. In order to examine the error caused by neglecting the breakage of dispersed drobbles, the cases of taking and not taking this factor into account have been simulated for an experimental column of 0.114 m diameter.

Figures 3a, b and c show the evolution of the density distribution function of drobbles based on the number along the column height without considering the breakage. These figures represent the growth characteristics of drobbles while the liquid phase of those drobbles gradually becomes vapour.

Clearly, neglecting the breakage of drobbles does not reflect the real situation. At $T_C - T_D = 10$ K the density distribution function still varies noticeably above 1.8 m. This means that the drobbles are still evaporating intensively above this height. However, the range of measured evaporation heights is only 0.35 to 0.70 m (see Fig. 7 or 8). This tells us that the breakage of drobbles in this system is very strong. Therefore, a more reasonable simulation requires us to include this factor in models.

Figure 4 demonstrates the simulated results of the density distribution function considering the breakage of drobbles under the same conditions as in Fig. 3. The curves below 0.18 m have a similar variation pattern to Fig. 3. The reason is that at a lower height of column the diameter of drobbles is so small that the probability of breakage of the drobbles is very low. Consequently, the evaporation progresses as if there was no breakage. Over a height of about 0.18 m the breakage of drobbles becomes stronger.

Breakage makes drobbles smaller and hence causes the curves of the density distribution function to move towards the left, while the growth of drobbles has a reverse effect. If a balance between the growth and the breakage of drobbles is reached, the curves will be stable (over a height of about 0.3 m in Fig. 4).

Figures 5 and 6 depict the density distribution function under a driving temperature difference of 2 K and 20 K, respectively. At $T_C - T_D = 2$ K the evaporation of drobbles goes so slowly that within a large range of height the variation pattern of the density distribution function is similar to that without considering the breakage of drobbles. At $T_C - T_D = 20$ K the evaporation proceeds much more rapidly and thus the breakage of drobbles becomes effective much earlier. Over a height of about 0.18 m, the density distribution function approaches a stable form.

Figure 7 shows a comparison of the evaporation height between experiment and calculation. Calculations are carried out taking the overall evaporation degree as 0.96. Note that "Experiment 1" and "Experiment 2" in the figure refer to two mass flow rates of the dispersed phase: 0.0333 kg/s and 0.0556 kg/s, respectively. The calculated evaporation height is much higher than the experimental data if the breakage of drobbles is not considered.

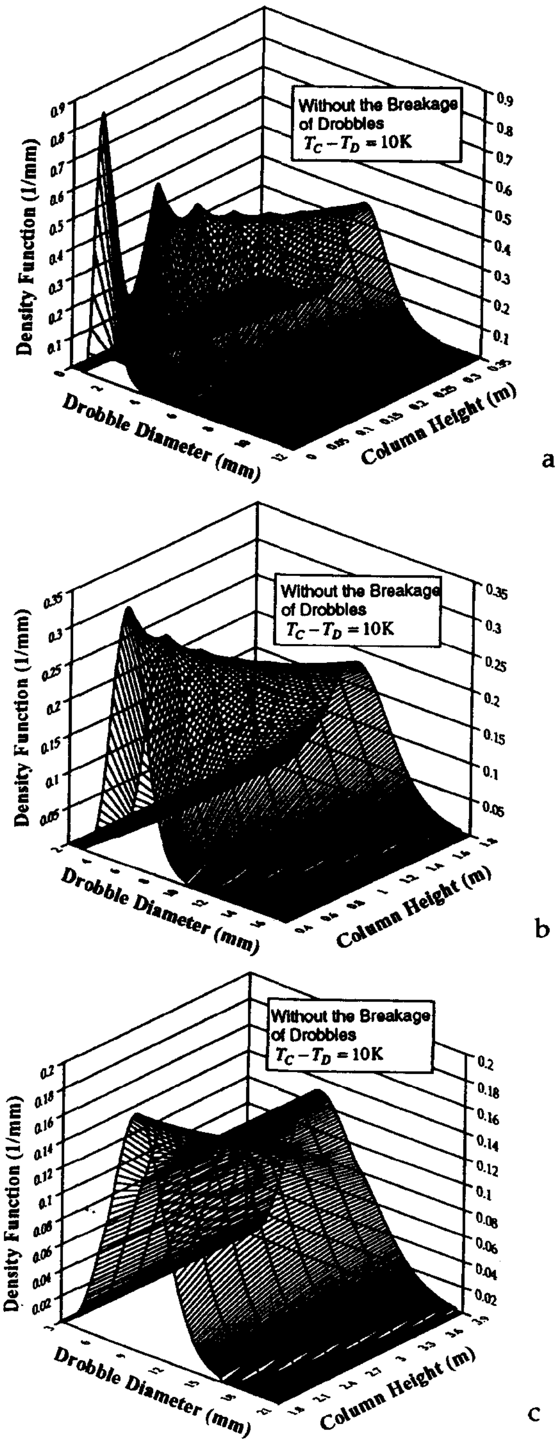

Fig. 3. Evolution of the density distribution function without considering the breakage of drobbles at $T_C - T_D = 10$ K.

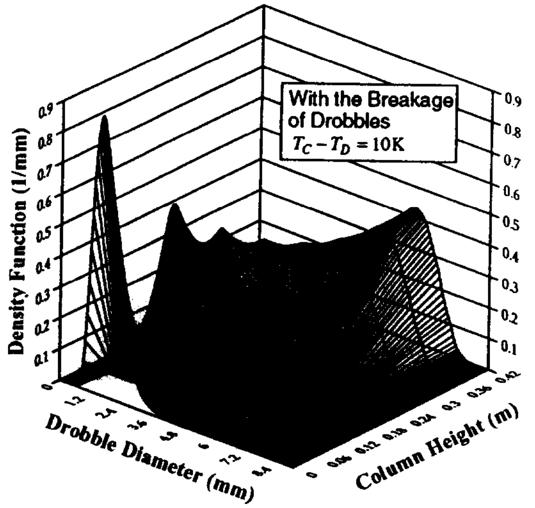

Fig. 4. Evolution of the density distribution function with considering the breakage of drobbles at $T_C - T_D = 10$ K.

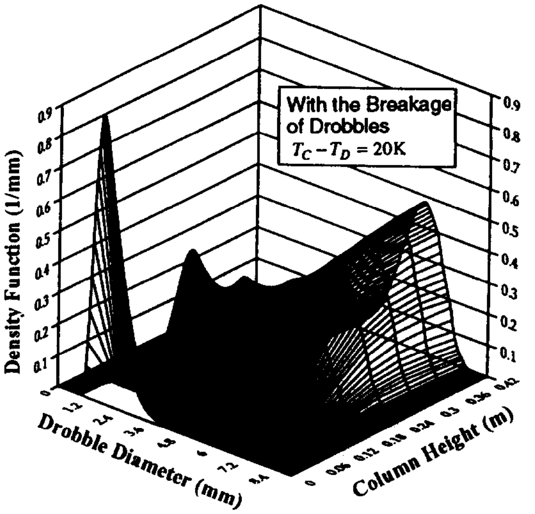

Fig. 6. Evolution of the density distribution function with considering the breakage of drobbles at $T_C - T_D = 20$ K.

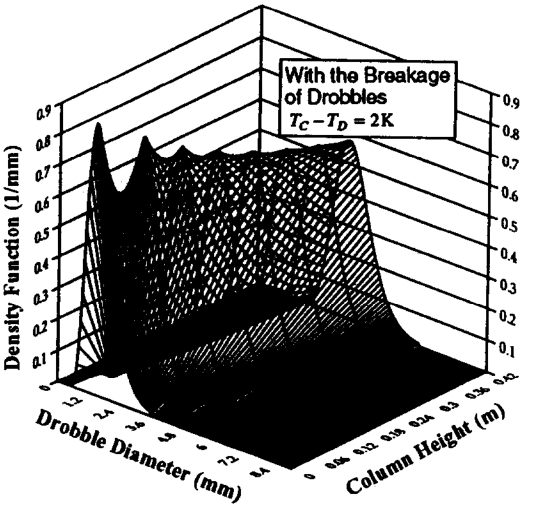

Fig. 5. Evolution of the density distribution function with considering the breakage of drobbles at $T_C - T_D = 2$ K.

Driving temperature difference

The driving force of evaporation is the temperature difference ΔT between the continuous and the dispersed phase. Consequently, it is one of the most important parameters influencing on the evaporation process. The influence of this parameter on the density distribution function and on the evaporation height were analyzed in the above section.

Many experimental data were obtained by Nitsche [22] and Steiner [10] under different temperature differences in bubble columns of different diameters. From experiments the evaporation height Δh decreases first very rapidly and then slowly as ΔT increases. If experimental data are drawn against the reciprocal of ΔT, an approximately linear line is produced with the help of the regressive fitting method. This line passes through the origin in this coordinate system since Δh should be zero at an infinite driving temperature difference. Nitsche [22] gave the following proportional relationship between Δh and ΔT in a column of 0.114 m diameter:

$$\Delta h \sim \Delta T^{-0.75}. \tag{17}$$

Figure 8 gives the simulation results of Δh at different temperature differences. The theoretical results are in good agreement with the experimental data from Steiner [10]. The calculated points can be joined in a nearly perfectly straight line. With the least square method the following proportional relationship is obtained

$$\Delta h \sim \Delta T^{-0.94}. \tag{18}$$

This result more closely approaches the conclusion made by Steiner, that is, there is a proportional relationship between the evaporation height and the reciprocal of the driving temperature difference.

Maximum diameter of stable drobbles

If a drop and/or bubble breaks up once its diameter reaches the maximum value given by Mersmann [27], the breakage probability function will take the form of a step function (curve 1 in Fig. 9). This does not represent the real breakage process very well due to the random characteristics of drobbles in the flow field. The breakage probability function is more reasonably assumed to be in the form of curve 2 in Fig. 9. This curve is drawn from the cumulative function of a normal distribution, which is symmetrical about $D = D_{\max}$. This means that a drop and/or bubble may break up even though its diameter is less than $D_{\max}$, and that it probably will not break up even though it has a diameter larger than this value. A drop and/or bubble of smaller diameter breaks up with a lower

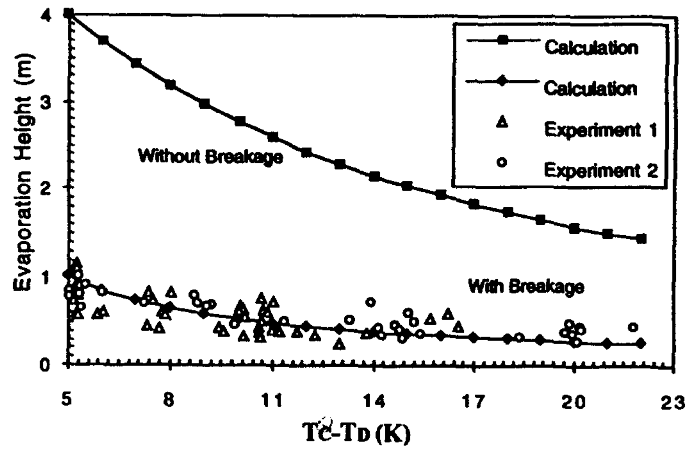

Fig. 7. Calculated evaporation height without and with taking the breakage of drobbles into account.

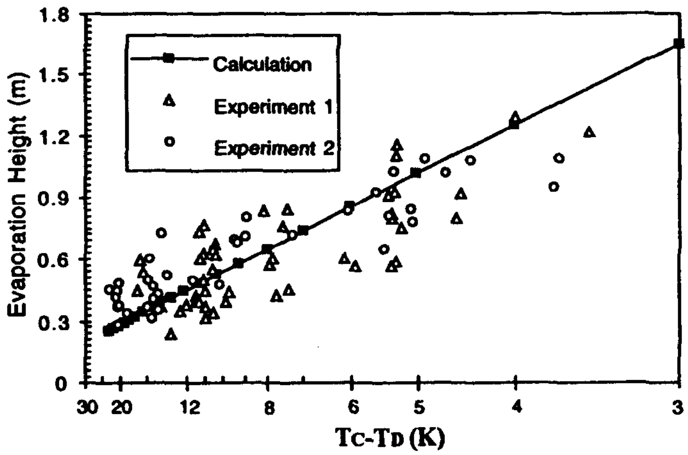

Fig. 8. Comparison of the evaporation height between simulation and experiment.

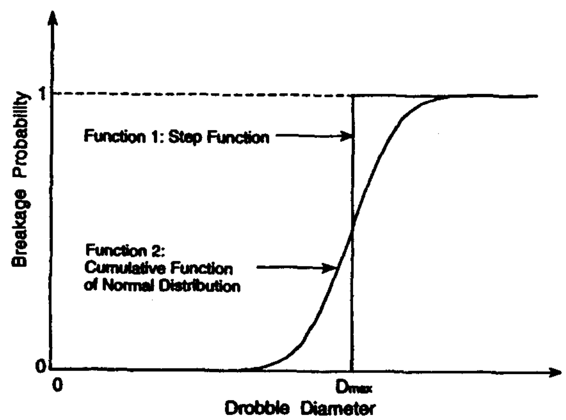

Fig. 9. Distribution functions of diameter of stable drobbles.

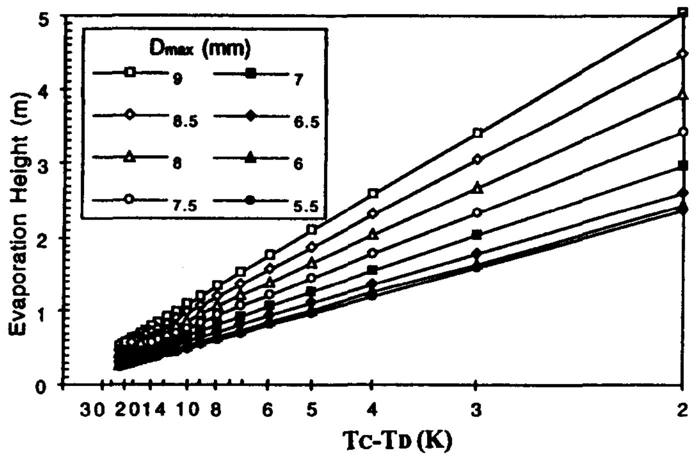

Fig. 10. Influence of the maximum diameter of stable drobbles on the evaporation height.

probability, and vice versa. Mersmann's formula represents the statistical average of a real process.

Figure 10 gives the calculation results of the evaporation height with different D_{max}. The simulation is carried out with the breakage probability function described by curve 2. A series of straight lines are formed in the coordinate system $(1/\Delta T, \Delta h)$. If the maximum diameter is less than 6.5 mm, the evaporation height varies very little. The direct relationship of Δh vs D_{max} is shown in Fig. 11.

Initial mean diameter of drops

The initial mean diameter of drops from the bottom distributor is also a very sensitive factor influencing the evaporation process. All simulated results given above were obtained with an initial diameter of 1.5 mm, which was used by Steiner [10] in term of his experimental distributor.

Figure 12 shows the evaporation height against the temperature difference at different initial mean diameters of drops. From this numerical simulation we can see that for a large initial mean diameter the evaporation height decreases rapidly at low temperature differences. This means that the initial mean diameter has a bigger influence on the evaporation height at a smaller driving temperature difference than at a larger one.

CONCLUDING REMARKS

From our simulation we can make the following conclusions:

1. The model established by the authors has a good capacity for simulation, with which we can examine the influence of various parameters on the evaporation process. The solution method, *split-*

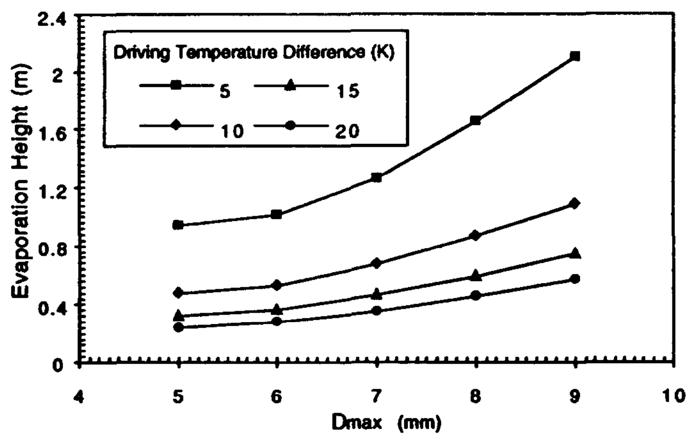

Fig. 11. Relationship of Δh vs D_{max}.

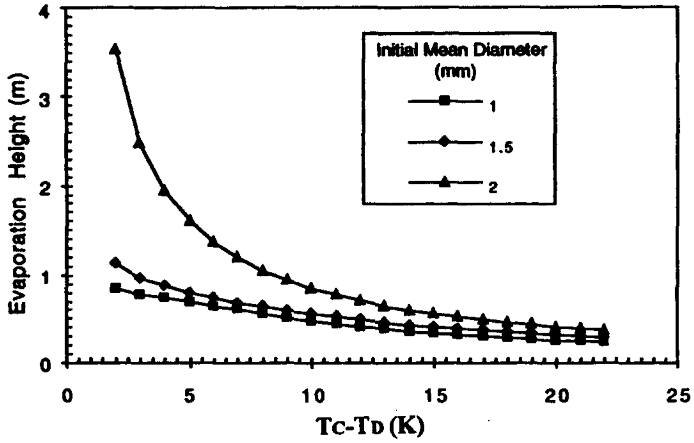

Fig. 12. Influence of the initial mean diameter of drops on the evaporation height.

ting + analog is very effective in solving the population balance equation with particle size growth. This method always leads to a stable calculation.

2. Little information is available to permit a definition of the functional form of the breakage function, $p(y, x)$, and the number of drobbles formed per breakage in bubble columns, $v(x)$. Binary breakage and a normal distribution seem to be two reasonable assumptions.
3. Although Mersmann's formula was obtained from single-phase drops or bubbles, it is also applicable to two-phase drobbles.
4. Numerical calculations show that an approximately proportional relationship exists between the evaporation height and the reciprocal of the driving temperature difference, confirming experiments performed by Steiner [10].

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